Name KEY Date________________ Period_____

Exponential Models

You will be using the exponential equation. There are two ways it can be written.

$y = a \cdot b^x$	$y = a \cdot (1 \,^+/\!- \ r)^x$
Where a = starting value and b = the rate of change	Where a = starting value and r = % increase or decrease

1.) Identify the initial amount, a, and the growth factor, b, in each exponential function:

a.) $g(x) = 20 \cdot 2^x$

a = 20
b = 2

b.) $y = 200 \cdot 1.0875^x$

a = 200
b = 1.0875

c.) $f(t) = 1.5^t$

a = ∅
b = 1.5

2.) Rewrite each increase as 1 + r .

Example: 55% *Answer→* 1 + .55

a.) 4%

1+.04

b.) 3.7%

1+.037

c.) 15%

1+.15

d.) 8.75%

1+.0875

3.) Rewrite each value as either 1 + r or 1 – r. Then state the rate of increase or decrease as a percent.

Example: 1.15 *Answer→* 1 + 0.15; rate of increase is 15%

a.) 1.75

1 + .75
rate of increase is 75%

b.) 0.85

1 − .15
rate of decrease is 15%

c.) 0.05

1 − .95
rate of decrease is 95%

d.) 3.6

1 + 2.6
rate of increase is 260%

4.) Identify each function as "exponential growth" or "exponential decay."

a.) $y = 0.68 \cdot 2^x$

exp growth
Because b is greater than 1

b.) $y = 2 \cdot 0.68^x$

exp. decay
Because b is less than 1

c.) $y = 68 \cdot 0.2^x$

exp. dec.
Because b is less than 1

5.) For each function, plug in values for x to find y to complete the data table.

a.) $y = 100 \cdot 0.9^x$

X	Y
0	100
1	90
2	81
3	72.9

b.) $y = 2 \cdot 10^x$

X	Y
0	2
1	20
2	200
3	2000

c.) $y = 10 \cdot 0.1^x$

X	Y
0	10
1	1
2	.1
3	.01

6.) Using your graphing calculator, type in the following data lists into your L1 and L2. Then click STAT→ CALC→0 to write an equation. Then find the missing values.

EQUATIONS:

a.) $y = 2.5(.3)^x$

X	Y
0	2.5
1	.75
3	.0675
5	.006075
8	1.64×10^{-4}
10	1.48×10^{-5}

b.) $y = 456(.7)^x$

X	Y
1	319.2
3	156.408
4	109.4856
6	53.647944
7	37.554
10	12.881

c.) $y = 75(.2)^x$

X	Y
2	3
4	.12
6	.0048
8	1.92×10^{-4}
10	7.68×10^{-6}

7.) Write an exponential function to model each situation. Tell what each variable you use represents.

a.) A population of 130, 000 grows 1% per year.

$y = 130{,}000(1 + .01)^x$

y represents the total population.
x represents the year.

b.) A price of $50 increases 6% each year.

$y = 50(1 + .06)^x$

y = total amount of money
x = year

c.) 3,000,000 initial population 1.5% annual decrease.

$y = 3000000(1 - .015)^x$

y = total population
x = year

Find the amount after 10 years.

$y = 3000000(1 - .015)^{10} =$

2579191

d.) A $900 purchase at 20% loss in value each year.

$y = 900(1-.2)^x$ y = total value, x = year

Find the value after 6 years.

$y = 900(1-.2)^6 = \$235.93$

8.) Would you rather have $500 in an account paying 6% interest compounded quarterly (or every 3 months) or $750 in an account paying 5.5% compounded annually (or every 12 months)? Explain your reasoning.

See Last Page

9.) The function $y = 355 \cdot 1.08^x$ models the average annual cost y (in dollars) for tuition and fees at public two-year colleges. The variable x represents the number of years since 1980.

Year	X	Y
1980	0	355
1985	5	521.61
1990	10	766.42
1995	15	1126.12

a.) What was the average annual cost in 1980?

$355

b.) What is the average percent increase in the annual cost?

8%

c.) Find the average annual cost in 1985, 1990, and 1995.

See Table.

d.) Find the average annual cost for the year you plan to graduate high school.

2011: $y = 355(1.08)^{31}$, $y = \$3858$

2010: $y = 355(1.08)^{30}$, $y = \$3572$

10.) In 1980, the population of Warren, Michigan, was about 161, 000. Since then the population has decreased about 1% per year.

a.) Write an equation to model the population of Warren since 1980.

$y = 161000(1-.01)^x$

b.) Using your equation, determine the population of Warren in 1990.

$y = 161000(1-.01)^{10}$

$y = 145606$

c.) Suppose the current trend continues. What will the population be in 2010?

$y = 161000(1-.01)^{30}$

$y = 119092$

Scientific Notation:

Write each number in scientific notation:		
1.) 0.0075 7.5×10^{-3}	**2.) $.85 \times 10^4$** 8.5×10^3	**3.) 0.65×10^{-2}** 6.5×10^{-3}

Write each number in standard notation:		
4.) 25×10^0 25	**5.) 658×10^{-3}** .658	**6.) 2.9×10^{-1}** .29

Write each answer in scientific notation:	
7.) $(5.4 \times 10^3)(6 \times 10^5)$ 3.24×10^9	**8.) $\frac{9.3 \times 10^{10}}{3.0 \times 10^5}$** 3.1×10^5

Use the properties of exponents to rewrite each expression:		
9.) $(a^4b^2)^2$ a^8b^4	**10.) $\frac{48x^5}{4x^3}$** $12x^2$	**11.) $\frac{15t^3v^4}{5tv^6}$** $\frac{3t^2}{v^2}$

Simplify each expression:	
12.) $(3x + 1) - (4x - 2)$ $3x + 1 - 4x + 2$ $-x + 3$	**13.) $(6x + 8)(x + 2)$** $6x^2 + 12x + 8x + 16$ $6x^2 + 20x + 16$

Problem #8:

Would you rather have $500 in an account paying 6% interest compounded quarterly (or every 3 months) or $750 in an account paying 5.5% compounded annually (or every 12 months)? Explain your reasoning.

Annual Interest Rate of 6%

Compounded Interest	Periods Per Year	Rates Per Period
Annually	1	6% every year
Semi-Annually	2	6% ÷ 2 = 3% every 6 months
Quarterly	4	6% ÷ 4 = 1.5% every 3 months
Monthly	12	6% ÷ 12 = .5% every month

Using the equation $y = a \cdot b^x$
x = the number of interest periods
y = the balance at various times

$500 in an account paying 6% interest compounded quarterly.

a = 500

$b = (1 + r) = 1 + \frac{.06}{4} = 1.015$

$y = 500(1.015)^x$

Year	Interest Periods	Total Balance
	X	Y
0	0	500
1	4	530.68
2	8	563.25
5	20	673.43
10	40	907.01
20	80	1645.33

$750 in an account paying 5.5% compounded annually.

a = 750

$b = (1 + r) = 1 + \frac{.055}{1} = 1.055$

$y = 750(1.055)^x$

Year	Interest Periods	Total Balance
	X	Y
0	0	750
1	1	791.25
2	2	834.77
5	5	980.22
10	10	1281.11
20	20	2188.32

So, in this case it is better to select the smaller annual interest rate with the larger starting value. The larger interest rate, when compounded quarterly does not benefit over the long term.